Solve with us 8 - Not Graded

**This assignment will not be graded and is only for practice.**

1.Key points:**1.Key points:**

* If x*x* approaches a*a* from the right side of a*a*, and the value of f*f* approaches lR*lR*​, then it is denoted by lim⁡x→a+f(x)=lR.*x*→*a*+lim​*f*(*x*)=*lR*​.
* If x*x* approaches a*a* from the left side of a*a*, and the value of f*f* approaches lL*lL*​, then it is denoted by lim⁡x→a−f(x)=lL.*x*→*a*−lim​*f*(*x*)=*lL*​.
* If lR=lL,*lR*​=*lL*​, then the limit at the point a*a* exists and it's value is lR=lL=l*lR*​=*lL*​=*l* (say).                                                                                                                                                                                lim⁡x→a−f(x)=lim⁡x→a+f(x)=lim⁡x→af(x)=l*x*→*a*−lim​*f*(*x*)=*x*→*a*+lim​*f*(*x*)=*x*→*a*lim​*f*(*x*)=*l*
* Definition: A function f*f* is continuous at a*a* if the limit of f*f* at a*a* exists and lim⁡x→af(x)=f(a).*x*→*a*lim​*f*(*x*)=*f*(*a*).                                                                                              Equivalently, f*f* is continuous at a*a* if and only if f(an)→f(a)*f*(*an*​)→*f*(*a*) whenever an→a*an*​→*a* }
* (Sandwich principle):**(Sandwich principle):** If lim⁡x→af(x)=L,lim⁡x→ag(x)=L*x*→*a*lim​*f*(*x*)=*L*,*x*→*a*lim​*g*(*x*)=*L*, and h(x)*h*(*x*) is a function such that f(x)≤h(x)≤g(x),*f*(*x*)≤*h*(*x*)≤*g*(*x*), then lim⁡x→ah(x)=L*x*→*a*lim​*h*(*x*)=*L*

***1 point***

Let f(x)*f*(*x*) be a function defined as f(x)=x2+1*f*(*x*)=*x*2+1. Which of the following statements is (are) true?

f*f* is defined at 22.

lim⁡x→2f(x)=4*x*→2lim​*f*(*x*)=4

f(2)=5*f*(2)=5

f*f* is continuous at 22

***1 point***

Let f*f* be a function and the Figure M2W2SU1 represent the graph of function f*f*. The solid points denote the value of the function at the points, and the values denoted by the hollow points are not taken by the functions.  
  
                                       A graph of a function

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Choose the set of correct options.  
[Hint: Try to find out the value of the function, as x*x* approaches a point from the left or the right. ]

lim⁡t→2−f(t)=10*t*→2−lim​*f*(*t*)=10

lim⁡t→2+f(t)=2*t*→2+lim​*f*(*t*)=2

lim⁡t→4−f(t)=4*t*→4−lim​*f*(*t*)=4

lim⁡t→4+f(t)=4*t*→4+lim​*f*(*t*)=4

f*f* is continuous at x=2*x*=2.

f*f* is continuous at x=6*x*=6

***1 point***

Define a function f*f* as follows: f(x)={xtan−12xif x>0bif x=0sin(ax)xif x<0,*f*(*x*)=⎩⎨⎧​*tan*−12*xx*​*bxsin*(*ax*)​​if *x*>0if *x*=0if *x*<0,​ Which of the following options is true if f*f* is continuous?  
[Hint:**Hint:** It is enough to check continuity at 0. Use lim⁡x→0xtan⁡−1x=1 and lim⁡x→0sin⁡xx=1*x*→0lim​tan−1*xx*​=1 and *x*→0lim​*x*sin*x*​=1 to compute left and right limit.]

a=b=2*a*=*b*=2

a=2*a*=2, b=12*b*=21​

a=12*a*=21​, b=2*b*=2

a=b=12*a*=*b*=21​

2. Key points:**2. Key points:**

* Differentiability:**Differentiability:** Let f*f* be a function defined on an interval around a*a*. Then f*f* is differentiable at a if lim⁡h→0f(a+h)−f(a)h*h*→0lim​*hf*(*a*+*h*)−*f*(*a*)​ exists.
* Differentiability implies continuity: **Differentiability implies continuity:** If f*f* is differentiable at a*a*, then it is continuous at a*a*.
* Checking differentiabilty by looking of the graph of a function: A function is not differentiable at a point if there is a sharp corner at that point, i.e., there is no tangent line, or there is a vertical tangent line.

***1 point***

Let f*f* be a function defined as f(x)=5x*f*(*x*)=5*x*. Which of the following options shows the step wise solution to check whether or not the function f*f* is differentiable at x=2*x*=2?

lim⁡h→0f(2+h)−f(2)h=lim⁡h→05(2+h)−10h=lim⁡h→05hh=5*h*→0lim​*hf*(2+*h*)−*f*(2)​=*h*→0lim​*h*5(2+*h*)−10​=*h*→0lim​*h*5*h*​=5.

lim⁡h→0f(0+h)−f(0)h=lim⁡h→05(0+h)−0h=lim⁡h→05hh=5*h*→0lim​*hf*(0+*h*)−*f*(0)​=*h*→0lim​*h*5(0+*h*)−0​=*h*→0lim​*h*5*h*​=5.

lim⁡h→0f(2+h)+f(2)h=lim⁡h→05(2+h)+10h=lim⁡h→020+5hh=∞*h*→0lim​*hf*(2+*h*)+*f*(2)​=*h*→0lim​*h*5(2+*h*)+10​=*h*→0lim​*h*20+5*h*​=∞, so function is not differentiable

lim⁡h→0f(2+h)h=lim⁡h→05(2+h)h=lim⁡h→010+5hh=10*h*→0lim​*hf*(2+*h*)​=*h*→0lim​*h*5(2+*h*)​=*h*→0lim​*h*10+5*h*​=10.

lim⁡h→0f(2+h)−f(2)h2=lim⁡h→05(2+h)−10h2=lim⁡h→05hh2=0*h*→0lim​*h*2*f*(2+*h*)−*f*(2)​=*h*→0lim​*h*25(2+*h*)−10​=*h*→0lim​*h*25*h*​=0.

***1 point***

Which of the following options shows the step wise solution to check whether or not the functions mentioned in the option is differentiable at the point mentioned in the option?  
[Hint:**Hint:** Use the definition of differentiability of a function and apply the concept of limit. ]

f(x)=a*f*(*x*)=*a*, at any real number c*c*: lim⁡h→0f(c+h)−f(c)h=lim⁡h→0a+h−ah=lim⁡h→0hh=1*h*→0lim​*hf*(*c*+*h*)−*f*(*c*)​=*h*→0lim​*ha*+*h*−*a*​=*h*→0lim​*hh*​=1.

f(x)=x−c*f*(*x*)=*x*−*c*, at c*c* for some real number c*c*: lim⁡h→0f(c+h)−f(c)h=lim⁡h→0(c+h−c)−(c−c)h=lim⁡h→0hh=1*h*→0lim​*hf*(*c*+*h*)−*f*(*c*)​=*h*→0lim​*h*(*c*+*h*−*c*)−(*c*−*c*)​=*h*→0lim​*hh*​=1.

f(x)=x2*f*(*x*)=*x*2, at any real number c*c*: lim⁡h→0f(c+h)−f(c)h=lim⁡h→0(c+h)2−c2h=lim⁡h→02ch+h2h=0*h*→0lim​*hf*(*c*+*h*)−*f*(*c*)​=*h*→0lim​*h*(*c*+*h*)2−*c*2​=*h*→0lim​*h*2*ch*+*h*2​=0

f(x)=ex*f*(*x*)=*ex*, at any real number c*c*: lim⁡h→0f(c+h)−f(c)h=lim⁡h→0ec+h−ech=lim⁡h→0ec(eh−1)h=eclim⁡h→0eh−1h=ec.1=ec*h*→0lim​*hf*(*c*+*h*)−*f*(*c*)​=*h*→0lim​*hec*+*h*−*ec*​=*h*→0lim​*hec*(*eh*−1)​=*ech*→0lim​*heh*−1​=*ec*.1=*ec*.

***1 point***

Consider the graphs given below:  
  
A graph of a function and a line

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Choose the set of correct options.

[**Hint**: Check whether they are continuous at each point in their domain, and also

check whether there is any sharp corner in the graphs. ]

f1*f*1​ is continuous and differentiable at each real number.

f1*f*1​ is not differentiable at 11.

f2*f*2​ is not continuous at 00.

f2*f*2​ is differentiable in the interval [1,2][1,2].

3. Key points:**3. Key points:**

* The Derivative of a function f*f* is denoted by f′(x)*f*′(*x*) and defined as f′(x)=lim⁡h→0f(x+h)−f(x)h*f*′(*x*)=*h*→0lim​*hf*(*x*+*h*)−*f*(*x*)​.
* Useful rules about derivatives

                   i) If f(x)*f*(*x*) and g(x)*g*(*x*) are differentiable at the point a*a*, then so is (f+g)(x)(*f*+*g*)(*x*), and (f+g)′(a)=f′(a)+g′(a)(*f*+*g*)′(*a*)=*f*′(*a*)+*g*′(*a*).  
                   ii) If f(x)*f*(*x*) is differentiable at the point a and c∈R*a* and *c*∈R, then (cf)(x)(*cf*)(*x*) is also differentiable at the point a*a* and (cf)′(a)=cf′(a)(*cf*)′(*a*)=*cf*′(*a*).  
                   iii) If f(x)*f*(*x*) and g(x)*g*(*x*) are differentiable at the point a*a*, then so is (f−g)(x)(*f*−*g*)(*x*), and (f−g)′(a)=f′(a)−g′(a)(*f*−*g*)′(*a*)=*f*′(*a*)−*g*′(*a*).  
                   iv)  The product rule:**The product rule:** If f(x)*f*(*x*) and g(x)*g*(*x*) are differentiable at the point a*a*, then so is (fg)(x)(*fg*)(*x*), and (fg)′(a)=f′(a)g(a)+f(a)g′(a)(*fg*)′(*a*)=*f*′(*a*)*g*(*a*)+*f*(*a*)*g*′(*a*).  
                   v)  The quotient rule:**The quotient rule:** If f(x)*f*(*x*) and g(x)*g*(*x*) are differentiable at the point a*a* and g(a)≠0*g*(*a*)=0, then so is fg(x)*gf*​(*x*), and (fg)′(a)=f′(a)g(a)−f(a)g′(a)g(a)2(*gf*​)′(*a*)=*g*(*a*)2*f*′(*a*)*g*(*a*)−*f*(*a*)*g*′(*a*)​.  
                   vi) Composition: the chain rule:**Composition: the chain rule:** If f(x)*f*(*x*) and g(x)*g*(*x*) are differentiable functions, then so is f(g(x))*f*(*g*(*x*)), and its derivative is : f(g(x))′=f′(g(x))g′(x)*f*(*g*(*x*))′=*f*′(*g*(*x*))*g*′(*x*).

* Derivatives of some specific functions:**Derivatives of some specific functions:**

             i) If f(x)*f*(*x*) is a function, then (cf(x))′=cf′(x)(*cf*(*x*))′=*cf*′(*x*).  
             ii) If f(x)*f*(*x*) is a function, then (f(cx))′=cf′(cx)(*f*(*cx*))′=*cf*′(*cx*).  
             iii) If f(x)=xn, then derivative of the function, f′(x)=nx(n−1)*f*(*x*)=*xn*, then derivative of the function, *f*′(*x*)=*nx*(*n*−1).  
             iv) If f(x)=sin⁡ax, then derivative of the function, f′(x)=acos⁡ax*f*(*x*)=sin*ax*, then derivative of the function, *f*′(*x*)=*a*cos*ax*.  
             v) If f(x)=cos⁡ax, then derivative of the function, f′(x)=−asin⁡ax*f*(*x*)=cos*ax*, then derivative of the function, *f*′(*x*)=−*a*sin*ax*.  
             vi) If f(x)=eax, then derivative of the function, f′(x)=aeax*f*(*x*)=*eax*, then derivative of the function, *f*′(*x*)=*aeax*.  
             vii) If f(x)=ln⁡ax, then derivative of the function, f′(x)=aax=1x*f*(*x*)=ln*ax*, then derivative of the function, *f*′(*x*)=*axa*​=*x*1​.  
             viii) If f(x)=log⁡bax, then derivative of the function, f′(x)=aaxln⁡b=1xln⁡b*f*(*x*)=log*b*​*ax*, then derivative of the function, *f*′(*x*)=*ax*ln*ba*​=*x*ln*b*1​.

* Indeterminate limits:**Indeterminate limits:** Suppose lim⁡x→af(x)=0 and lim⁡x→ag(x)=0*x*→*a*lim​*f*(*x*)=0 and *x*→*a*lim​*g*(*x*)=0 or both diverge to ∞∞ or both diverge to =−∞=−∞ then the lim⁡x→af(x)g(x)*x*→*a*lim​*g*(*x*)*f*(*x*)​ is said to be an indeterminate limit.
* L’Hopital’s rule:**L’Hopital’s rule:** In the situation of an indeterminate limit, suppose the following conditions holds:                                                                                                                              i)] f′(x) and g′(x)*f*′(*x*) and *g*′(*x*) exist on the interval I*I* containing a*a* (except possibly at a*a*).                                                                                                                    ii)] g′(x)≠0*g*′(*x*)=0 in the interval I*I*.                                                                                                                                                                                                             iii)] lim⁡x→af′(x)g′(x)=L*x*→*a*lim​*g*′(*x*)*f*′(*x*)​=*L*.                                                                                                                                                                                                                                              Then lim⁡x→af(x)g(x)=L*x*→*a*lim​*g*(*x*)*f*(*x*)​=*L*.

***1 point***

If f(x)=x5+sin⁡3x+2cos⁡4x+e3x+log⁡23x*f*(*x*)=*x*5+sin3*x*+2cos4*x*+*e*3*x*+log2​3*x*, then f′(x)=*f*′(*x*)=  
[Hint:**Hint:** Use the derivative formulas of specific functions ]

5x4+3cos⁡3x−8sin⁡4x+3e3x+1xln⁡25*x*4+3cos3*x*−8sin4*x*+3*e*3*x*+*x*ln21​

5x5+3cos⁡3x−8sin⁡4x+3e3x+1xln⁡25*x*5+3cos3*x*−8sin4*x*+3*e*3*x*+*x*ln21​

5x4+3cos⁡3x+8sin⁡4x+3e3x+1xln⁡25*x*4+3cos3*x*+8sin4*x*+3*e*3*x*+*x*ln21​

5x4+3cos⁡3x−8sin⁡4x+e3x+1xln⁡25*x*4+3cos3*x*−8sin4*x*+*e*3*x*+*x*ln21​

5x4+3cos⁡3x−8sin⁡4x+3e3x+1x5*x*4+3cos3*x*−8sin4*x*+3*e*3*x*+*x*1​

***1 point***

In which of the following options can one apply L'Hopital's rule to evaluate the limits?  
[Hint:**Hint:** First check which of the following functions satisfy the criteria to apply L'Hopital's rule. ]

lim⁡x→0xx+sin⁡x*x*→0lim​*x*+sin*xx*​

lim⁡x→0sin⁡2x1−cos⁡(2x)*x*→0lim​1−cos(2*x*)sin2*x*​

lim⁡x→07+ln⁡xx3+6*x*→0lim​*x*3+67+ln*x*​

lim⁡x→0+1x*x*→0+lim​*x*1​

***1 point***

Consider a function f(x)=logx5*f*(*x*)=log*x*​5, where x>0*x*>0, and x≠1*x*=1. Derivative of f*f* is  
[Hint:**Hint:** We know log⁡x5=1log⁡5xlog*x*​5=log5​*x*1​, then use quotient rule of derivative on specific function 1log⁡5xlog5​*x*1​. ]

−1x2ln⁡5(log⁡5x)2−*x*2ln5(log5​*x*)21​

−1xln⁡5(log⁡5x)2−*x*ln5(log5​*x*)21​

1xln⁡5(log⁡5x)2*x*ln5(log5​*x*)21​

1x2ln⁡5(log⁡5x)2*x*2ln5(log5​*x*)21​

4. Key points:**4. Key points:**

* In the XY−*XY*−plane, the equation of a line joining two points (x1,y1) and (x2,y2)(*x*1​,*y*1​) and (*x*2​,*y*2​) is y−y1=y2−y1x2−x1(x−x1)*y*−*y*1​=*x*2​−*x*1​*y*2​−*y*1​​(*x*−*x*1​).

* Equation of the tangent at a point on a curve: **Equation of the tangent at a point on a curve:** Let f(x)*f*(*x*) be differentiable at the point a*a*. Then the tangent to f*f* at a*a* exists and is given by

                                                                   y=f′(a)(x−a)+f(a).*y*=*f*′(*a*)(*x*−*a*)+*f*(*a*).

* Suppose the tangent to f*f* at a*a* exists and it is not vertical (i.e is not the line x=a*x*=*a*). Then f*f* is differentiable at a*a* and hence the equation of the tangent is

                                                                y=f′(a)(x−a)+f(a).*y*=*f*′(*a*)(*x*−*a*)+*f*(*a*).

* If f(x)*f*(*x*) is differentiable at a*a*, then the best linear approximation is given by Lf(x)=f(a)+f′(a)(x−a)*Lf*​(*x*)=*f*(*a*)+*f*′(*a*)(*x*−*a*).

* If there is a best linear approximation for f*f* at a*a*, then f*f* is differentiable at a*a* and hence equation of best linear approximation is given by Lf(x)=f(a)+f′(a)(x−a)*Lf*​(*x*)=*f*(*a*)+*f*′(*a*)(*x*−*a*).

***1 point***

What will be the equation of the tangent of a parabola given by f(x)=4x2*f*(*x*)=4*x*2 at the point (2,16)(i.e., x=2)(2,16)(*i*.*e*., *x*=2)?  
[Hint:**Hint:** Differentiate the function f(x)*f*(*x*) at x=2*x*=2 and use the tangent formula y=f′(2)(x−2)+f(2)*y*=*f*′(2)(*x*−2)+*f*(2). ]

y=16(x−1)*y*=16(*x*−1)

y=8x+8*y*=8*x*+8

y=16x*y*=16*x*

y=4x*y*=4*x*

***1 point***

Consider the function f:R→R*f*:R→R, such that f(x)=2x+5*f*(*x*)=2*x*+5. Which of the following expression represents the best linear approximation Lf(x)*Lf*​(*x*) at the origin?  
[Hint:**Hint:** Differentiate the function f(x)*f*(*x*) at x=0*x*=0 and use the best linear approximation formula formula Lf(x)=f′(0)(x−0)+f(0)*Lf*​(*x*)=*f*′(0)(*x*−0)+*f*(0). ]

Lf(x)=5*Lf*​(*x*)=5

Lf(x)=2x*Lf*​(*x*)=2*x*

Lf(x)=2x+5*Lf*​(*x*)=2*x*+5

Lf(x)=0*Lf*​(*x*)=0

Let f(x)*f*(*x*) be differentiable at x=1*x*=1 and f*f* be represented by the curve C*C*. If a line which passes through the point (5,8)(5,8), is the tangent to the curve C*C* at the point (1,0)(1,0), then find the value of f′(1)*f*′(1).  
[Hint:**Hint:** Find the equation of the line using formula y−y1=y2−y1x2−x1(x−x1)*y*−*y*1​=*x*2​−*x*1​*y*2​−*y*1​​(*x*−*x*1​) and compare with the equation of tangent at x=1*x*=1 which is y=f′(1)(x−1)+f(1)*y*=*f*′(1)(*x*−1)+*f*(1). ]